Casimir Effect for Moving Branes in Static dS$_{4+1}$ Bulk

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In this paper we study the Casimir effect for conformally coupled massless scalar fields on background of Static dS$_{4+1}$ spacetime. We will consider the general plane–symmetric solutions of the gravitational field equations and boundary conditions of the Dirichlet type on the branes. Then we calculate the vacuum energy-momentum tensor in a configuration in which the boundary branes are moving by uniform proper acceleration in static de Sitter background. Static de Sitter space is conformally related to the Rindler space, as a result we can obtain vacuum expectation values of energy-momentum tensor for conformally invariant field in static de Sitter space from the corresponding Rindler counterpart by the conformal transformation.

1. INTRODUCTION

The Casimir effect is regarded as one of the most striking manifestation of vacuum fluctuations in quantum field theory. The presence of reflecting boundaries alters the zero-point modes of a quantized field, and results in the shifts in the vacuum expectation values of quantities quadratic in the field, such as the energy density and stresses. In particular, vacuum forces arise acting on constraining boundaries. The particular features of these forces depend on the nature of the quantum field, the type of spacetime manifold and its dimensionality, the boundary geometries and the specific boundary conditions imposed on the field. Since the original work by Casimir in 1948 (Casimir, 1948) many theoretical and experimental works have been done on this problem (see, e.g., Mostepanenko and Trunov, 1997; Plunien *et al.*, 1986; Lamoreaux, 1999; The Casimir Effect, 1999; Bordag *et al.*, 2001; Kirsten, 2001; Bordag, 2002; Milton, 2002) and references therein). There are several methods to calculate Casimir energy. For instance, we

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can mention mode summation, Green's function method (Plunien *et al.*, 1986), heat kernel method (Kirsten, 2001) along with appropriate regularization schemes such as point separation (Christensen, 1976, 1978; Adler *et al.*, 1977) dimensional regularization (Deser *et al.*, 1976, see also Capper and Duff, 1974, 1975), zeta function regularization (Hawking, 1977; Blau *et al.*, 1988; Elizalde *et al.*, 1994; Elizalde, 1995; Bytsenko *et al.*, 2003). Recently a general new methods to compute renormalized one–loop quantum energies and energy densities are given in Graham *et al.* (2002a, 2002b) (see also Elizalde, 2003).

The Casimir effect can be viewed as a polarization of vacuum by boundary conditions. Another type of vacuum polarization arises in the case of an external gravitational fields (Birrel and Davies, 1982; Grib *et al.*, 1994). It is well known that the vacuum state for an uniformly accelerated observer, the Fulling–Rindler vacuum (Fulling, 1973, 1977; Unruh, 1976; Boulware, 1975; Avagyan *et al.*, 2002), turns out to be inequivalent to that for an inertial observer, the familiar Minkowski vacuum. Quantum field theory in accelerated systems contains many of special features produced by a gravitational field avoiding some of the difficulties entailed by renormalization in a curved spacetime. In particular, near the canonical horizon in the gravitational field, a static spacetime may be regarded as a Rindler–like spacetime. Rindler space is conformally related to the static de Sitter space and to the Robertson–Walker space with negative spatial curvature. As a result the expectation values of the energy–momentum tensor for a conformally invariant field and for corresponding conformally transformed boundaries on the de Sitter and Robertson–Walker backgrounds can be derived from the corresponding Rindler counterpart by the standard transformation (Birrel and Davies, 1982). The authors in Birrel and Davies (1982) have been shown that the Minkowski vacuum contains a thermal spectrum of Rindler particles. One can also demonstrate this by showing that the Green functions in Minkowski vacuum are Rindler thermal Green functions. In a similar way one can relate the vacua of static de Sitter space and de Sitter space have the same curvature, but static de Sitter space is a member of Rindler class, while de Sitter space is a member of Minkowski space.

The past few years witnessed a growing interest among particle physicists and cosmologists toward models with extra space-like dimensions. This interest was initiated by string theorists (Witten, 1996; Horava and Witten, 1996; Banks and Dine, 1996), who exploited a moderately large size of an external 11th dimension in order to reconcile the Planck and string/GUT scales. Taking this idea further, it was shown that large extra dimensions allow for a reduction of the fundamental higher-dimensional gravitational scale down to the TeV-scale (Arkani-Hamed *et al.*, 1998, 1999; Antoniadis *et al.*, 1998). An essential ingredient of such a scenario is the confinement of the standard model fields on field theoretical defects, so that only gravity can access the large extra dimensions. These models are argued to make contact with an intricate phenomenology, with a variety of consequences for collider searches, low-energy precision measurements, rare decays

and astroparticle physics and cosmology. An alternative solution to the hierarchy problem was proposed in Randall and Sundrum (1999). This higher dimensional scenario is based on a non-factorizable geometry which accounts for the ratio between the Planck scale and weak scales without the need to introduce a large hierarchy between fundamental Planck scale and the compactification scale. The model consists of a spacetime with a single S^1/Z_2 orbifold extra dimension. In this context, the Casimir energy arising between the two static boundaries has been computed in Fabinger and Hořava (2000), Mirabelli and Peskin (1998), in the first of these two works, the backreaction on the geometry was taken into account. The same problem has been considered in five-dimensional anti-deSitter space in Nojiri *et al*. (2000), Brevik *et al*. (2001). Soon, the generalization of an AdS, flat or dS brane in the AdS bulk (Kachru *et al*., 2000), and of a flat or dS brane in dS bulk were studied carefully (Ito, 2002a). The localization of gravity in these models has also been discussed (Ito, 2002b). The bulk Casimir effect for a conformal or massive scalar when the bulk represents five-dimensional AdS or dS space with two or one four-dimensional dS brane, has been considered in Elizalde *et al*. (2003a) (see also Elizalde and Quiroga Hurtado, 2004; Elizalde *et al*., 2003b; Cognola *et al*., 2003a, 2003b). The recently proposed cyclic model of the universe (Khoury *et al*., 2001; Steinhardt and Turok, 2002) is also based on this framework in which the motion and collision of two such branes is responsible for the Big-Bang of the standard cosmology.

Recent astronomical observations of supernovae and cosmic microwave background (Riess *et al*., 1998; Perlmutter *et al*., 1999; de Bernardis *et al*., 2000; Bennett *et al*., 2003; Tegmark *et al*., 2003) indicate that the universe is accelerating and can be well approximated by a world with a positive cosmological constant. If the universe would accelerate indefinitely, the standard cosmology leads to an asymptotic dS universe. De Sitter spacetime plays an important role in the inflationary scenario, where an exponentially expanding approximately dS spacetime is employed to solve a number of problems in standard cosmology. In this paper we are interested in studying the possible effects of the Casimir energy in an scenario like the one mentioned before in which two branes are moving by uniform acceleration through the static de Sitter vacuum. The complete analysis of the problem is in general too involved to obtain explicit analytic results and, for that reason, we will consider a simplified model in which the two branes are perfectly flat, ignoring possible gravitational effects. In any realistic model of a brane collision process it will be necessary to consider the acceleration and the brane curvature (Rasanen, 2002). To see similar model in which the two branes are moving with constant relative velocity refer to (Maroto, 2003), as the author of this refrence have been mentioned "the present analysis would be the first (velocity-dependent) correction to the flat static case" then may be could say that our model is second (accelerated -dependent) correction to the static case.

This problem for the conformally coupled Dirichlet and Neumann mass-less scalar and electromagnetic fields in four dimensional Rindler spacetime was considered by Candelas and Deutsch (1977). Investigation of local physical characteristics in the Casimir effect, such as expectation value of the energy-momentum tensor, is of considerable interest. In addition to describing physical structure of the quantum field at a given point, the energy-momentum tensor acts as the source in the Einstein equations and therefore plays an important role in modeling a self-consistent dynamics involving the gravitational field. Here we will investigate the vacuum expectation values of the energy-momentum tensor for the massless scalar field with conformal curvature coupling and satisfying Dirichlet boundary condition on the infinite plane in five spacetime dimension. Here we use the results of Avagyan *et al.* (2002) to generate vacuum energy–momentum tensor for the static de Sitter background which is conformally related to the Rindler spacetime. Previously this method has been used in Setare and Mansouri (2001) to drive the vacuum stress on parallel plates for scalar field with Dirichlet boundary condition in de Sitter space. Also this method has been used in Saharian and Setare (2003) to derive the vacuum characteristics of the Casimir configuration on background of conformally flat brane-world geometries for massless scalar field with Robin boundary condition on plates.

2. VACUUM EXPECTATION VALUES FOR THE ENERGY-MOMENTUM TENSOR

In this paper we will consider a conformally coupled massless scalar field $\varphi(x)$ satisfying the equation

$$\left(\nabla_\mu \nabla^\mu + \frac{3}{16} R\right) \varphi(x) = 0, \tag{1}$$

on background of a dS_{4+1} spacetime. In Eq. (1) ∇_μ is the operator of the covariant derivative, and R is the Ricci scalar for the corresponding metric g_{ik}. In static coordinates $x^i = (t, r, \theta, \theta_2, \phi)$, dS metric has the form

$$ds_{\mathrm{dS}}^2 = g_{ik} dx^i \, dx^k = \left(1 - \frac{r^2}{\alpha^2}\right) dt^2 - \frac{dr^2}{1 - \frac{r^2}{\alpha^2}} - r^2 \, d\Omega_3^2, \tag{2}$$

where $d\Omega_3^2$ is the line element on the 3–dimensional unit sphere in the Euclidean space, and the parameter α defines the dS curvature radius. Note that $R = 12/\alpha^2$. Our main interest in the present paper is to investigate the vacuum expectation values (VEV's) of the energy–momentum tensor for the field $\varphi(x)$ in the background of the above de Sitter spacetime induced by two parallel plates moving with uniform proper acceleration. we will consider the case of a scalar field satisfying

Dirichlet boundary condition on the surface of the plates:

$$\varphi \mid_{\xi=\xi_1} = \varphi \mid_{\xi=\xi_2} = 0. \tag{3}$$

The presence of boundaries modifies the spectrum of the zero–point fluctuations compared to the case without boundaries. This results in the shift in the VEV's of the physical quantities, such as vacuum energy density and stresses. This is the well known Casimir effect.

First of all let us present the dS line element in the form conformally related to the Rindler spacetime. With this aim we make the coordinate transformation $x^i \rightarrow x'^i = (\tau, \xi, \mathbf{x}'), \mathbf{x}' = (x'^2, x'^3, x'^4)$ (see Birrel and Davies, 1982, for the case $3 + 1$-dimensional case)

$$\tau = \frac{t}{\alpha}, \quad \xi = \frac{\sqrt{\alpha^2 - r^2}}{\Omega}, \quad x'^2 = \frac{r}{\Omega} \sin\theta \, \cos\theta_2,$$

$$x'^3 = \frac{r}{\Omega} \sin\theta \, \sin\theta_2 \, \cos\phi, \quad x'^4 = \frac{r}{\Omega} \sin\theta \, \sin\theta_2 \, \sin\theta_2 \, \sin\phi, \tag{4}$$

with the notation

$$\Omega = 1 - \frac{r}{\alpha} \cos\theta. \tag{5}$$

Under this coordinate transformation the dS line element takes the form

$$ds^2_{\mathrm{dS}} = g'_{ik} dx'^i \, dx'^k = \Omega^2 \left(\xi^2 d\tau^2 - d\xi^2 - d\mathbf{x}'^2 \right). \tag{6}$$

In this form the dS metric is manifestly conformally related to the Rindler spacetime with the line element ds^2_{R}:

$$ds^2_{\mathrm{dS}} = \Omega^2 ds^2_{\mathrm{R}}, \quad ds^2_{\mathrm{R}} = g_{\mathrm{R}ik} dx'^i \, dx'^k = \xi^2 d\tau^2 - d\xi^2 - d\mathbf{x}'^2, \quad g'_{ik} = \Omega^2 g_{\mathrm{R}ik}. \tag{7}$$

The Casimir effect with boundary conditions (3) on two parallel plates moving with uniform proper acceleration on background of the Rindler spacetime is investigated in Avagyan *et al.* (2002) for a scalar field with a Dirichlet and Neumann boundary condition. The expectation values of the energy-momentum tensor for a scalar field $\varphi_{\mathrm{R}}(x')$ in the Fulling-Rindler vacuum can be presented in the form of the sum

$$\langle 0_R | T_i^k[g_{\mathrm{R}lm}, \varphi_{\mathrm{R}}] | 0_R \rangle = \langle \tilde{0}_R | T_i^k[g_{\mathrm{R}lm}, \varphi_{\mathrm{R}}] | \tilde{0}_R \rangle + \langle T_i^k[g_{\mathrm{R}lm}, \varphi_{\mathrm{R}}] \rangle^{(b)}, \tag{8}$$

where $|0_R\rangle$ are $|\tilde{0}_R\rangle$ are the amplitudes for the vacuum in the Rindler space in presence and absence of the branes respectively, $\langle T_i^k[g_{\mathrm{R}lm}, \varphi_{\mathrm{R}}]\rangle^{(b)}$ is the part of the vacuum energy-momentum tensor induced by the branes. In the case of a conformally coupled massless scalar field for the part without boundaries one has

$$\langle \tilde{0}_R | T_i^k[g_{\mathrm{R}lm}, \varphi_{\mathrm{R}}] | \tilde{0}_R \rangle = \frac{\delta_i^k}{32\pi^2 \xi^5} \int_0^\infty \frac{\omega^4 d\omega}{e^{2\pi\omega} + 1} \left(\frac{1}{4\omega^2} + 1 \right). \tag{9}$$

For a scalar field $\varphi_R(x')$, satisfying the Dirichlet boundary condition, the boundary induced part in the region between hypersurface have the form (Avagyan *et al.*, 2002)

$$\langle T_i^k[g_{Rlm}, \varphi_R]\rangle^{(b)} = A_4 \delta_i^k \int_0^\infty dk k^4 \int_0^\infty d\omega \left\{ \frac{\sinh \pi\omega}{\pi} f^{(i)}[\tilde{D}_{i\omega}(k\xi, k\xi_2)] \right.$$
$$\left. - \frac{I_\omega(k\xi_1)}{I_\omega(k\xi_2)} \frac{F^{(i)}[D_\omega(k\xi, k\xi_2)]}{D_\omega(k\xi_1, k\xi_2)} \right\}, \quad (10)$$

where

$$A_4 = \frac{1}{4\pi^{5/2}\Gamma(3/2)}. \quad (11)$$

Also we have introduced the notation

$$\tilde{D}_{i\omega}(k\xi, k\xi_2) = K_{i\omega}(k\xi) - \frac{K_{i\omega}(k\xi_2)}{I_{i\omega}(k\xi_2)} I_{i\omega}(k\xi), \quad (12)$$

and the functions $F^{(i)}[G(z)]$, $i = 0, \ldots, 4$ are as following

$$F^{(i)}[G(z)] = f^{(i)}[G(z), \omega \to i\omega]. \quad (13)$$

Here for a given function $G(z)$ we use the notations

$$f^{(0)}[G(z)] = \frac{1}{8}\left|\frac{dG(z)}{dz}\right|^2 + \frac{3}{16z}\frac{d}{dz}|G(z)|^2 + \frac{1}{8}\left[1 + 7\frac{\omega^2}{z^2}\right]|G(z)|^2, \quad (14)$$

$$f^{(1)}[G(z)] = -\frac{1}{2}\left|\frac{dG(z)}{dz}\right|^2 - \frac{3}{16z}\frac{d}{dz}|G(z)|^2 + \frac{1}{2}\left(1 - \frac{\omega^2}{z^2}\right)|G(z)|^2, \quad (15)$$

$$f^{(i)}[G(z)] = -\frac{|G(z)|^2}{3} + \frac{1}{8}\left[\left|\frac{dG(z)}{dz}\right|^2 + \left(1 - \frac{\omega^2}{z^2}\right)|G(z)|^2\right];$$
$$i = 2, 3, 4 \quad (16)$$

where $G(z) = D_{i\omega}(z, k\xi_2)$, which given by following expression, and the indices 0,1 correspond to the coordinates τ, ξ respectively,

$$D_{i\omega}(k\xi, k\xi_2) = I_{i\omega}(k\xi_2)K_{i\omega}(k\xi) - K_{i\omega}(k\xi_2)I_{i\omega}(k\xi). \quad (17)$$

To find the vacuum expectation values generated by the branes in the dS$_{4+1}$ space, first we will consider the corresponding quantities in the coordinates $(\tau, \xi, \mathbf{x}')$ with the metric (6). The latters are found by using the standard transformation formula for the conformally related problems:

$$\langle 0_{dS}|T_i^k[g'_{lm}, \varphi]|0_{dS}\rangle = \Omega^{-5}\langle 0_R|T_i^k[g_{Rlm}, \varphi_R]|0_R\rangle + \langle T_i^k[g'_{lm}, \varphi]\rangle^{(an)}, \quad (18)$$

where the second summand on the right is determined by the trace anomaly and is related to the divergent part of the corresponding effective action by the relation (Birrel and Davies, 1982)

$$\langle T_i^k [g'_{lm}, \varphi] \rangle^{(an)} = 2g'^{kl} \frac{\delta}{\delta g'^{il}(x)} W_{\text{div}}[g'_{mn}, \varphi]. \tag{19}$$

Note that in odd spacetime dimensions the conformal anomaly is absent and the corresponding anomaly part vanishes:

$$\langle T_i^k [g'_{lm}, \varphi] \rangle^{(an)} = 0. \tag{20}$$

The formulae given above allow us to present the dS vacuum expectation values in the form similar to (8):

$$\langle 0_{\text{dS}} | T_i^k [g'_{lm}, \varphi] | 0_{\text{dS}} \rangle = \langle \tilde{0}_{\text{dS}} | T_i^k [g'_{lm}, \varphi] | \tilde{0}_{\text{dS}} \rangle + \langle T_i^k [g'_{lm}, \varphi] \rangle^{(b)}, \tag{21}$$

where $\langle \tilde{0}_{\text{dS}} | T_i^k [g'_{lm}, \varphi] | \tilde{0}_{\text{dS}} \rangle$ are the vacuum expectation values in the dS space without boundaries and the part $\langle T_i^k [g'_{lm}, \varphi] \rangle^{(b)}$ is induced by the branes. Conformally transforming the Rindler results one finds

$$\langle \tilde{0}_{\text{dS}} | T_i^k [g'_{lm}, \varphi] | \tilde{0}_{\text{dS}} \rangle = \Omega^{-5} \langle \tilde{0}_{\text{R}} | T_i^k | \tilde{0}_{\text{R}} \rangle + \langle T_i^k [g'_{lm}, \varphi] \rangle^{(an)}, \tag{22}$$

$$\langle T_i^k [g'_{lm}, \varphi] \rangle^{(b)} = \Omega^{-5} \langle T_i^k [g_{\text{R}lm}, \varphi_{\text{R}}] \rangle^{(b)}. \tag{23}$$

Under the conformal transformation $g'_{ik} = \Omega^2 g_{\text{R}ik}$, the field φ_{R} will change by the rule

$$\varphi(x') = \Omega^{-3/2} \varphi_{\text{R}}(x'), \tag{24}$$

where the conformal factor is given by expression (5). The vacuum expectation values of the energy-momentum tensor in coordinates are obtained from expressions (22) and (23) by the standard coordinate transformation formulae. As before, we will present the corresponding components in the form of the sum of purely dS and boundary parts:

$$\langle 0_{\text{dS}} | T_i^k [g_{lm}, \varphi] | 0_{\text{dS}} \rangle = \langle \tilde{0}_{\text{dS}} | T_i^k [g_{lm}, \varphi] | \tilde{0}_{\text{dS}} \rangle + \langle T_i^k [g_{lm}, \varphi] \rangle^{(b)}. \tag{25}$$

By using the relations (4) between the coordinates for the purely dS part one finds

$$\langle \tilde{0}_{\text{dS}} | T_i^k [g_{lm}, \varphi] | \tilde{0}_{\text{dS}} \rangle = \frac{(\alpha^2 - r^2)^{-\frac{5}{2}}}{32\pi^2 \Gamma(2) \xi^5} \int_0^\infty \frac{\omega^4 d\omega}{e^{2\pi\omega} + 1}$$

$$\times \left(\frac{1}{4\omega^2} + 1 \right) \text{diag} \left(-1, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right). \tag{26}$$

This formula generalizes the result for $3 + 1$-dimension given, for instance, in Birrel and Davies (1982). As the for boundary induced energy-momentum tensor

the spatial part is not isotropic, the corresponding part in the coordinates x^i is more complicated:

$$\langle T_i^k [g_{lm}, \varphi] \rangle^{(b)} = \Omega^{-5} \langle T_i^k [g_{Rlm}, \varphi_R] \rangle^{(b)}, \quad i, k = 0, 3, 4, \tag{27}$$

$$\langle T_1^1 [g_{lm}, \varphi] \rangle^{(b)} = \frac{(\cos \theta - r/\alpha)^2}{\Omega^7} \langle T_1^1 [g_{Rlm}, \varphi_R] \rangle^{(b)}$$

$$+ \frac{1 - r^2/\alpha^2}{\Omega^5} \sin^2 \theta \langle T_2^2 [g_{Rlm}, \varphi_R] \rangle^{(b)}, \tag{28}$$

$$\langle T_1^2 [g_{lm}, \varphi] \rangle^{(b)} = \frac{(r/\alpha - \cos \theta) \sin \theta}{r \Omega^7} \left\{ \langle T_1^1 [g_{Rlm}, \varphi_R] \rangle^{(b)} \right.$$

$$\left. - \langle T_2^2 [g_{Rlm}, \varphi_R] \rangle^{(b)} \right\}, \tag{29}$$

$$\langle T_2^2 [g_{lm}, \varphi] \rangle^{(b)} = \frac{1 - r^2/\alpha^2}{\Omega^7} \sin^2 \theta \langle T_1^1 [g_{Rlm}, \varphi_R] \rangle^{(b)}$$

$$+ \frac{(r/\alpha - \cos \theta^2)}{\Omega^7} \langle T_2^2 [g_{Rlm}, \varphi_R] \rangle^{(b)}, \tag{30}$$

where the expressions for the components of the boundary induced energy-momentum tensor in the Rindler spacetime are given by formula (10)–(16). As we see the resulting energy-momentum tensor is non-diagonal.

In the discussion above we have considered the vacuum energy-momentum tensor of the bulk. For a scalar field on manifolds with boundaries in addition to the bulk part the energy-momentum tensor contains a contribution located on the boundary. For arbitrary bulk and boundary geometries the expression of the surface energy-momentum tensor is given in Saharian (2003). In the case of a conformally coupled scalar field the transformation formula forthe surface energy-momentum tensor under the conformal rescaling of the metric is the same as that for the volume part. For our problem in this paper, the surface energy-momentum tensor is obtained from the corresponding Rindler counterpart by a way similar to that described above. The expression for the latter is given in Saharian (2003).

3. CONCLUSION

Over the last few years a lot of interest has been raised on the possibility that our universe is a $3-$brane embedded in a higher dimensional spacetime. Ordinary matter fields are assumed to live on the brane while gravity propagates in the whole spacetime. The main part of the work done in this direction refers to the branes sitting at a prescribed point of an extra dimension. However, it is tempting, even inspired by $D - p$-brane models, to consider that the three-brane is somehow let to move in the bulk.

In the present paper we have investigated the Casimir effect for a conformally coupled massless scalar field between two boundary branes moving by uniform acceleration, on background of the five-dimensional static de Sitter spacetime. We have assumed that the scalar field satisfies Dirichlet boundary condition on the branes. The static de Sitter spacetime is conformally related to the Rindler spacetime, then the vacuum expectation values of the energy-momentum tensor are derived from the corresponding Rindler spacetime results by using the conformal properties of the problem. The vacuum expectation value of the energy-momentum tensor for a brane in dS spacetime consists of two parts given in Eq. (21). The first one corresponds to the purely dS contribution when the boundary is absent. It is determined by formula (22), where the second term on the right is due to the trace anomaly and is zero for odd spacetime dimensions. The second part in the vacuum energy-momentum tensor is due to the imposition of boundary conditions on the fluctuating quantum field. The corresponding components are related to the vacuum energy-momentum tensor in the Rindler spacetime by Eqs. (27)–(30) and the Rindler tensor is given by formulae (10)–(13). Unlike to the purely dS part, the boundary induced part of the energy-momentum tensor is non-diagonal and depends on both dS static coordinates r and θ.

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